Determining the optimal product mix in multiple constraints manufacturing environment: an application in the textile industry

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ABSTRACT – REZUMAT

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Theory of constraints (TOC) is an approach to production planning and control by focusing on the constraints of an organization to increase throughput by effectively managing constraints. TOC approach has been applied in many sectors, and efficient results have been taken. One of the application areas of TOC is product mix decisions. Product mix decisions are important for multi-product manufacturing systems because they affect the performance measures of the companies. This study aims to present how TOC is applied to determine the optimum product mix in a multiple-constraint environment in the textile industry. To achieve this, we first select three basic products of a textile company and examine the production processes of these products from TOC perspective. Next, we perform a bottleneck process and identify three bottlenecks for the problem. Then, based on our bottleneck process results, we generate three scenarios. Upon assessing these scenarios, we determine the most appropriate product mix by implementing the TOC approach. Finally, we employ a goal programming approach to solve the product mix problem and compare its results with those obtained by the TOC.

Keywords: *theory of constraints, product mix, multiple constraints, goal programming, textile industry, Turkey*

Determinarea combinației optime de produse în mediul de producție cu constrângeri multiple: o aplicație în industria textilă

Teoria constrângerilor (TOC) este o abordare a planificării și controlului producției prin concentrarea pe constrângerile unei organizații, pentru a crește randamentul prin gestionarea eficientă a acestora. Abordarea TOC a fost aplicată în multe sectoare și s-au obținut rezultate eficiente. Unul dintre domeniile de aplicare ale TOC este reprezentat de deciziile legate de combinația de produse. Deciziile privind combinația de produse sunt importante pentru sistemele de producție, deoarece afectează măsurile de performanță ale companiilor. Acest studiu își propune să prezinte modul în care se aplică TOC pentru a determina combinația optimă de produse într-un mediu cu constrângeri multiple din industria textilă. Pentru a realiza acest lucru, selectăm mai întâi trei produse de bază ale unei companii textile și examinăm procesele de producție din perspectiva TOC. Apoi, efectuăm un proces de blocaj și identificăm trei blocaje pentru această problemă. Apoi, pe baza rezultatelor procesului nostru de blocaj, generăm trei scenarii. După evaluarea acestor scenarii, determinăm cea mai potrivită combinație de produse prin implementarea abordării TOC. În cele din urmă, folosim o abordare de programare a obiectivelor pentru a rezolva problema combinației de produse și a compara rezultatele acesteia cu cele obținute prin TOC.

Cuvinte-cheie*: teoria constrângerilor, combinația de produse, constrângeri multiple, programarea obiectivelor, industria textilă, Turcia*

INTRODUCTION

The theory of constraints (TOC) is a business management approach that evaluates firms as a system and states that each system has at least one constraint and that the power of the systems is as much as the strength of its constraint. For profitability, these constraints must be correctly identified, managed, and monitored continuously concerning different criteria [1].

Product mix decisions, which have a significant impact on the profitability of the company, are one of the most important decisions faced by production companies [2]. The product mix problem contains determining the quantity of each product within the product portfolio of the company. The main structure of the problem is to maximize profit from the mix of manufactured products linked to constraints on the available capacity of resources [3].

The product mix decision problem is one of the important applications of the TOC's ongoing improvement process. Extensive research has been performed to determine the best product mix for profit maximization [2]. A company has at least one constraint which limits the company from achieving the best performance and maximum profit. A constraint is defined as "a thing that prevents making money of a system/organization” [4]. When a company has capacity constraints, capacity is limited, it can't manufacture every unit of product that the market demands. In this case, the best action is to concentrate on the company's most profitable products and to utilize all of the company's available resources to produce these products. As a result, the company will be able to boost its profitability by utilizing its existing resources to develop the most profitable products [5].

The TOC approach is a widely used method to determine the product mix of a company. This approach was used in many sectors such as the TFT-LCD industry [6], furniture sector [7], birds' food production sector [8], etc. When the literature was investigated, there is not any study conducted in the textile industry on the product mix problem. Therefore, one aim of this study is to show how the TOC approach can be used in the textile industry to determine product mix.

Most of the studies related to product mix problems have used data provided from previous studies such as de Soza et al. proposing an approach [9] and testing this approach on data provided by Fredendall and Lea [5]. Sobreiro and Nagano evaluated the heuristics of Fredendall and Lea [10] and Aryanezhad and Komijan [11] and proposed a new heuristic [12]. Tanhaei and Nahavandi [13] used a goal programming approach for the product mix problem defined by Hsu and Chung [14]. Another aim of this study is to use real-world data provided by a textile company.

In the literature, many studies about product mix were performed in a single constraint environment. There exist only a few studies that incorporate multi-constraints [13, 15–17]. Another aim of this study is to contribute to the TOC applications performed in multi-constraint environments.

In this study, our main ambition is to investigate the impact of identifying and eliminating constraints that arise during the production process in terms of profitability. To achieve this, the application of a production company performing in the textile industry was questioned to determine whether (*i*) there exist constraints that limit the effectiveness of the company, (*ii*) these constraints that arise during the production process can be eliminated, and (*iii*) these constraints will affect the profitability of the company. In this study, we use the TOC methodology and goal programming approach in the context of a multi-constraint manufacturing environment.

GENERAL INFORMATION

The basic argument of TOC is that constraints determine the performance of a firm, and each system has at least one constraint [18]. TOC is a management approach defending which constraints must be eliminated because of limiting the performance of enterprises, and that constraints have negative impacts on performance. TOC is a systematic approach focused on the identification and elimination of constraints for continuous development [19].

Goldratt defined a simple Five Focusing Steps (5FSs) process for achieving continuous improvement. These five steps are explained in detail in the literature [2, 19]. TOC's 5FS are as follows:

1. Identify the system's constraint(s).
2. Decide how to exploit the system's constraint(s).
3. Subordinate everything else to the above decision.
4. Elevate the constraint(s).
5. If, in the previous steps, a constraint has been broken, go back to Step 1.

There are different approaches in the literature regarding the classification of constraints. According to Louderback and Patterson [20], constraints are divided into two groups internal constraints and external constraints. Internal constraints are production capacity, operating policies, and the working environment. External constraints are market share, legal restrictions, etc. [20].

The new performance criteria are improved in cost distribution by changing cost and management conceptions in enterprises. Performance measures are divided into two groups; financial measures and operational measures. While financial measures are net profit, investment profitability, and cash flow, operational measures are throughput, inventory, and operating expenses [21].

Net Profit (NP): Net profit is an absolute measure of whether the firm makes money or not.

Investment Profitability (ROI): Investment profitability is a proportional measure of a firm's target of earning money.

Cash flow: The amount of money available for the company to meet its financial obligations.

Throughput (T): It is the money rate the firm gains through sales. Goldratt described throughput as the difference between the sales price of the unit product and the direct first material cost.

Inventory (I): Inventory “represents the whole money that the firm deposit to things bought to sell.” Unlike other approaches, in TOC, inventory is described as an entity, not as a source. Inventories are evaluated by the cost of the raw materials. Labour costs and general production expenses are not included in variable costs. According to this, buildings, and vehicles are included in product and semi-finished product inventories [22, 23].

Operating Expenses (OE): Operating expenses represent the whole money that the firm spends to transform inventory into a product. In TOC, expenses are defined according to sales volume, not to production volume. Operating expenses include general administrative expenses, direct labour costs, general production expenses, marketing, sales, and distribution costs.

The product mix problem is widely acknowledged as one of the most critical decision problems of a production system. It is not possible to meet the demand for all items due to capacity limits.

Therefore, companies need to decide on the appropriate quantities of suitable products to participate in the production plan to achieve the desired profit [24]. The issue of product mix includes deciding the volume and mix of products to maximize profit within constraints of production resources and the capacity of constraints. Although the integer linear programming method can optimize the product mix, it is not always easy and fast to formulate and solve a mathematical model [25]. The product mix is an NP-hard problem because of the complexity of the product mix decision problem.

The TOC-based approach is frequently used in place of or in addition to optimization tools such as the contribution margin per constraint unit method or linear programming approaches as a tool for product mix selections. Many articles published since the early 1990s have been used in several similar examples to analyse the quality of the TOC-based approach for possible product mix decisions compared to other tools [26]. Table 1 provides a literature review of the product mix based on the TOC approach.

Table 1

LITERATURE SUMMARY ABOUT PRODUCT MIX BASED ON THE TOC APPROACH

Author(s) of the study	The main scope of the study
Onwubolu	A heuristic approach to a Tabu search-based TOC to identify the nearly optimal product mix for minor problems [27].
Onwubolu and Muting	A TOC procedure based on a genetic algorithm to solve product mix problems [28].
Lea and Fredendall	Effects of production performance on management accounting systems and methods of determining product mix [29].
Aryanezhad and Komijan	The TOC-heuristic method with Linear Programming, an algorithm that determines the product mix (TOC-AK) [11].
Mishra et al.	A taboo search and simulated annealing hybrid approach to determine the product mix in a multi-constraint environment [30].
Souren, Ahn and Schmitz	Several samples with modifications of the same basic sample, investigate optimal product mix decisions using a TOC-based approach [26].
Wang, Du and Wen	Mixed Integer Linear Programming to define a product mix for the TFT-LCD industry, taking into account profit, efficiency, raw material supply, and market demand [6].
Chaharsooghi and Jafari	A simulated annealing algorithm to determine the product mix [31].
Hasuike and Ishii	A flexible mix of problems using TOC and an efficient solution method using two stochastic programming models, namely the probability fractional optimization model and the probability maximization model [32].
Wang, Sun and Yang	An optimization approach based on an immunity algorithm and TOC for product mix on problems of small-scale or large-scale samples (100 items and 50 resources) [33].
Ray, Sarkar, and Sanyal	The combined use of TOC and analytical hierarchy process in product mix problems [16].
Susanto and Bhattacharya	Negotiated fuzzy multipurpose linear programming approach to determine the product mix of an eight-product chocolate production company by assuming the objective coefficients with fuzzy numbers [34].
de Soza et al.	An algorithm that determines the initial solution based on the RTOC presented by Fredendall and Lea [9].
Sobreiro and Nagano	Evaluated the heuristics of Fredendall and Lea [31] and Aryanezhad and Komijan [22] and proposed a new and better constructive heuristic based on the TOC and the Backpack Problem [12].
Tanhaei and Nahavandi	The improved TOC approach determines the optimal product mix in a two-constraint resource environment [15].
Badri, Ghazanfari, Shahanaghi	The product mix problem with range parameters and proposed a multi-criteria decision-making approach to determine the TOC-based product mix [25].
Sobreiro, Mariano and Nagano	A throughput per day approach to define product mix by a constructive heuristic based on Integer Linear Programming and heuristics-based in TOC [10].
Golmohammadi and Mansouri	A new mixed-integer programming (MIP) model by considering product mix problem and scheduling simultaneously (COLOMAPS) [35].
Okutmus, Kahveci and Kartašova	The constraint-based resource utilization approach to determine the optimal product mix in the furniture sector [7].
Tanhaie and Nahavandi	A methodology using of goal programming and pair-wise comparison to determine the product mix of the production system in multiple bottlenecks environment [13].
Zhuang and Chang	A mixed-integer programming (MIP) model, based on the time-driven activity-based costing (TDABC) accounting system [36].
Mohammed and Kassam	A Model using two linear programming models based on TOC to determine the product mix of a bird’s food production facility [8].
Wang et al.	They clarified the cases under which the AHP (Analytical Hierarchy Process) / TOC method can and cannot output the optimal solution in multi constraint environment [17].

METHOD AND MATERIAL

Method

The product mix was determined based on multiple objectives which were maximizing throughput and maximizing bottlenecks exploitation. Considering TOC's 5FS, to determine the optimal, the following steps should be performed [10, 37]:

1. Identify the system's constraint(s) by calculating the necessary capacity in each source to manufacture all products: the only constraint of the system is resources that its market demand to increase in capacity or its available capacity is smaller than its requested capacity.
2. Decide how to explore the system's constraint(s): the constraint should be explored by observing the throughput of each product for each consumed unit time of the constraint of the system.

The first, the TOC algorithm was suggested by Tanhaei and Nahavandi [15]. For the second step, the TOC algorithm and goal programming (GP) approach explained by Tanhaei and Nahavandi [13] were used in this study. The structural framework of the study is presented in figure 1.

Notations used in the methodology are presented in table 2.

Table 2

PARAMETERS	
Notation	**Explanation**
i	Product index i = 1, 2, …, n
j	Resource index j = 1, 2, ..., m
k	Bottleneck index k = 1, 2, …, r
r	Number of bottlenecks
X_i	Production quantity of product (unit)
RM_i	Raw material cost of product *I* (৳/ unit)
D_i	Demand of product *i* (unit)
P_i	Selling price of product *I* (৳/unit)
CP_j	Capacity of resource *j* (min.)
L_j	Load (Required capacity) of resource *j* (min.)
d_j	Difference between available capacity and required capacity of the resource (min.)
t_{ij}	Unit production time of product i in resource *j* (min./unit)
BN_k	Bottlenecks
T_i	The throughput of product *i* (৳/unit)
$T_{i(B1k)}$	The throughput per constraint resource time of BNk (৳/unit*min.)
NP	Net profit (৳/unit)
OE	Operating Expenses (৳)
G_s	Objective functions for maximizing bottlenecks exploitation
G_{r+1}	Objective function for maximizing throughput
W_s	Importance of objective function
f_s	Positive deviation from goal *s*
s	Number of objective functions

TOC based product mix methodology

Steps of the TOC-based product mix methodology adapted from Tanhaei and Nahavandi [15] for multiple constraints are presented as follows.

Step 1. Identifying system constraints.

Calculate the total load for each resource as follows:

$$L_j = \sum_{i=1}^{m} D_i t_{ij} \tag{1}$$

Then calculate d_j which is the difference between a resource's capacity (CP_j) and its total load as follows:

$$d_j = CP_j - L_j \tag{2}$$

where If $d_j \leq 0$ or d_j has a negative value and this resource has overload, it is a constraint, otherwise it is non-constraint. Determine the set of constraint resources (*CR*). Here *BN*1 is the main constraint, *BN*2 is the second constraint, etc. *CR* = {*BN*1,…, *BNk*}

Step 2. Decide how to exploit the system's constraint(s). Determine the throughput for each product i is determined as follows.

$$T_i = P_i - RM_i \tag{3}$$

Then, determine the throughput per constraint resource time of *BN*1 for each product *i* as follows:

$$T_{iBN1} = \frac{T_i}{t_{iBN1}} \qquad i = 1, 2, ..., n \tag{4}$$

determine the production priority of products by sequencing products according to T_{iBN1} descending. Determine the production quantity of products (X_i) at the point of demand of each product and the available capacity of *BN*1. Calculate the load on *BNk*+1 for the product mix determined as follows:

$$L_{BNk+1} = \sum_{i=1}^{r} t_{BNk+1}\, x_i \tag{5}$$

If $L_{BNk+1} \leq CP_{BNk+1}$, the product mix determined is optimal and stopped. If $L_{BNk+1} > CP_{BNk+1}$, Repeat Steps 2 for *BNk*+1. Determine the throughput per constraint resource time of *BNk*+1 for each product *i* as follows:

$$T_{(iBNk+1)-} = \frac{T_i}{t_{iBNk+1}} \qquad i = 1, 2, ..., n \tag{6}$$

repeat Step 2 for *BNk*+1.

Calculate the profit of the company according to the determined product mix as follows:

$$NP = \sum_{i=1}^{n} (T_i x_i - OE_i) \tag{7}$$

Goal programming model

GP Model for the product mix suggested by Tanhaei and Nahavandi [13] is explained in the following. Equations 8 and 9 show how the product mix model maximizes bottleneck utilization and throughput 9. Equation 10 determines that the total process time of all products at resource *j* does not exceed resource *j*'s capacity, and equation 11 determines that the output amount of product *i* does not exceed the product *i*'s demand.

$$G_s = max\left(\sum_{i=1}^{n} x_i t_{ij}\right) \tag{8}$$

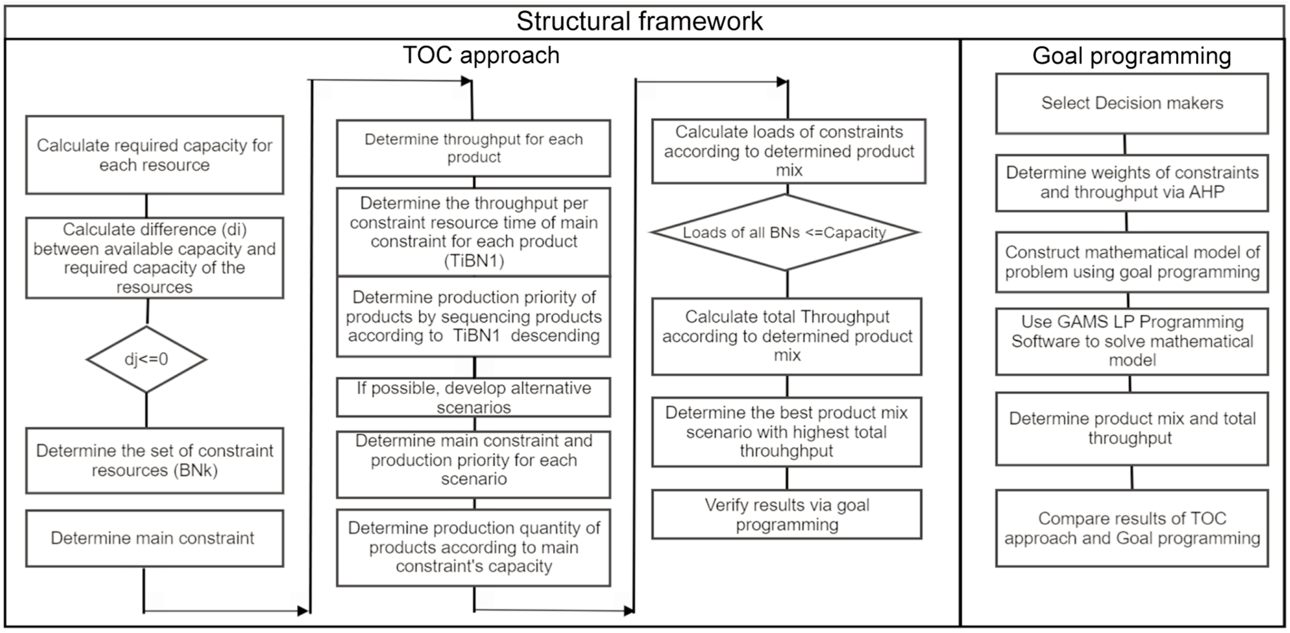

Fig. 1. The structural framework of the study

$$G_s = max\left(\sum_{i=1}^{n} x_i\, t_{ij}\right) \qquad (9)$$

Subject to

$$\sum_{i=1} L_j \le CP_j, \quad j = 1, 2, \dots, m \qquad (10)$$

$$0 \le x_i \le D_i \qquad (11)$$

The model is solved by using GP, and fs refers to deviations from targets. Equation 12 demonstrates how deviation from objectives could be reduced by explicitly including the positive deviation in the model's objective function, that is, it means minimizing the sum of the deviations from the targets. W_s demonstrates the weights of objectives that should be ascribed to variable f_s as determined by the decision-maker. W_s values can be determined via AHP.

Hence, the final model is given as follows: equation 13 determines the usage of constraints in possible maximum capacity. Equation 14 determines achieving to possible maximum throughput. Equation 15 determines that the total process time of all products at non-constraint resources. Equation 16 represents, the production quantity of product *i* does not exceed its demand.

$$min\left(\sum_{s=1}^{r+1} W_s\, f_s\right) \qquad (12)$$

Subject to

$$\sum_{i=1}^{n} x_i\, t_{ij} + f_s = CP_s, \quad j, s = 1, 2, \dots, r \qquad (13)$$

$$\sum_{i=1}^{n} x_i\, L_i + f_{r+1} = \sum_{i=1}^{n} D_i\, T_i \qquad (14)$$

$$\sum_{i=1}^{n} x_i\, t_{ij} \le CP_j \quad if \;\; r < j < m \qquad (15)$$

$$0 \le x_i \le D_i \qquad (16)$$

Material

XYZ Company mainly manufactures three kinds of products including suits, jackets, and trousers in the Istanbul factory in Turkey. The firm manufactures about 19 products in different concepts as outsourced to the firms in domestic and abroad. At the same time, it also keeps under the control of the global brand products of Turkey, the Middle East, Africa, and Russian markets. The unit sales price of the suit, jacket and trousers are ₺700, ₺500 and ₺250 respectively while their weekly demand is 700, 800 and 600 units.

Cost information of the products manufactured in the factory is presented in table 3. According to TOC, all costs except direct raw materials and supplies are considered operating expenses. Direct raw material expenses of the products, direct labour expenses, operating expenses, sales prices, and demands of the products are given in table 3.

In XYZ Company, the production process contains eight steps which are Mold/Model Preparation, Slaughterhouse, Fusing/Labelling, Jacket Production, Pants Production, Ironing, Quality Control, and Mapping/Packaging. The processing times of each product in production processes are given in table 3.

RESULTS

Identifying the system constraints

Firstly, loads of resources are calculated, and then loads and available capacities are compared to identify the constraints. The required capacity of resources (L_j) to produce 700 suits, 800 jackets, and 600 trousers was calculated, and then, the difference (d_j) between the resource's available capacity and its required capacity is calculated. Resources which is $L_j > CP_j$ are determined as constraints or bottlenecks. As can be seen from table 4, the company has three constraints: Slaughterhouse (*BN*1), Model Preparation (*BN*2), and Ironing (*BN*3). The company will not be able to meet customer demands, because of these constraints. Slaughterhouse is the main

Table 3

UNIT PROCESSING TIME IN PRODUCTION PROCESSES AND COST INFORMATION OF THE PRODUCTS			
Processes	**Suit**	**Jacket**	**Trousers**
Model preparation (min/unit)	2.05	1.26	0.79
Slaughterhouse (min/unit)	2.01	1.54	0.47
Fusing/Labelling (min/unit)	0.99	0.65	0.34
Sewing I (min/unit)	1.54	1.54	-
Sewing II (min/unit)	1.23	-	1.23
Ironing (min/unit)	1.84	1.3	0.54
Quality control (min/unit)	1.5	0.95	0.55
Combining/Packaging (min/unit)	1.4	0.8	0.6
Direct raw materials expenses (₺/unit)	220	133	72
Direct labour expenses ((₺/unit)	67.2	44.5	32.0
Variable operating expenses (₺/unit)	134.3	79.3	55.0
Fixed operating expenses (₺)	119,000.0	75,250.0	41,750.0
Sales price (₺/unit)	700	500	250
Demand (unit)	700	800	600

constraint with the largest d_j value (521) and the bottleneck. Firstly, the company should focus on Slaughterhouse (*BN*1). But Model Preparation (*BN*2) has very close values with Slaugherhouse (*BN*1), we have two alternatives to manage constraints.

Decide how to exploit the system's constraints

At this step, firstly, unit throughputs per product (T_i) are calculated and then throughput rates per constraint times ($Ti_{(BN1)}$) for products are calculated based on *BN*1. $Ti_{(BN1)}$ values are ordered in descending order. Thus production priorities of products are defined as Trousers, Suit, and Jacket accordingly based on $Ti_{(BN1)}$ values as seen in table 5.

Because $Ti_{(BN1)}$ values of Suit and Jacket are very close values, we have second alternative production priority as Trousers, Jacket and Suit.

According to tables 4 and 5, the above explanations, we determined three scenarios. Based on Scenario 1, the main constraint is Slaughterhouse (*BN*1), and the Production priority is Trousers, Suit, and Jacket. Based on Scenario 2, the main constraint is Slaughterhouse (*BN*1), and the Production priority is Trousers, Jacket and Suit. Based on Scenario 3, the main constraint is Model Preparation (*BN*2), Production priority is Jacket, Suit, and Trousers which are determined in table 6.

For three Scenarios, product mixes are determined as seen in table 7. The manufacturing quantity of the

Table 4

DETERMINING SYSTEM CONSTRAINTS								
Parameter	**Model preparation**	**Slaughter house**	**Fusing/ Labelling**	**Sewing I**	**Sewing II**	**Ironing**	**Quality control**	**Combining/ Packaging**
CP_j (min.)	2400	2400	2400	2400	2400	2400	2400	2400
L_j (min.)	2917	2921	1417	2310	1599	2652	2140	1980
d_j (min.)	-517	-521	983	90	801	-252	260	420
CP Rate	1.215	1.217	0.590	0.962	0.666	1.105	0.892	0.825
Bottleneck	BN2	BN1	-	-	-	BN3	-	-

Table 5

PRODUCTION PRIORITY OF PRODUCTS BASED ON BN1						
Product (i)	**P_i (₺/unit)**	**RM_i (₺/unit)**	**T_i (₺/unit)**	**$t_{i(BN1)}$ (min.)**	**$T_{i(BN1)}$ (₺/unit*min.)**	**Production priority**
Suit	700	220	480	2.01	238.806	2
Jacket	500	133	367	1.54	238.312	3
Trousers	250	72	178	0.47	378.723	1

Table 6

PRODUCTION PRIORITY OF PRODUCTS BASED ON BN2 FOR SCENARIO 3						
Product (i)	**P_i (₺/unit)**	**RM_i (₺/unit)**	**T_i (₺/unit)**	**$t_{i(BN2)}$ (min.)**	**$T_{i(BN2)}$ (₺/unit*min.)**	**Production priority**
Suit	700	220	480	2.01	234.15	2
Jacket	500	133	367	1.54	291.27	1
Trousers	250	72	178	0.47	225.32	3

Table 7

DETERMINING PRODUCT MIX FOR THREE SCENARIOS						
Parameter		**Suit**	**Jacket**	**Trousers**	**Capacities**	Demand: Suit: 700 unit Jacket: 800 unit Trousers: 600 unit
Unit processing Times (min/unit)	BN1	2.01	1.54	0.47	2400 min.	
	BN2	2.05	1.26	0.79	2400 min.	
	BN3	1.84	1.3	0.54	2400 min.	

Scenario 1			
Main Constaint: BN1 Priority 1. Trousers 2. Suits 3. Jacket	Required capacity for 600 Trousers (min)	600 × 0.47 = 282	Product Mix Trousers: 600 unit Suits: 700 unit Jacket: 461.7 unit
	Left capacity of BN1 for suits (min.)	2400 – 282 = 2118	
	Required capacity for 700 suits (min.)	700 × 2.01 = 1407	
	Left capacity of BN1 for jacket (min.)	1407 – 711 = 711	
	Jacket Quantity (unit)	711/1.54 = 461.7	
Scenario 2			
Main Constaint: BN1 Priority: 1. Trousers 2. Jacket 3. Suit	Required capacity for 600 Trousers(min.)	600 × 0.47 = 282	Product Mix Trousers: 600 unit Jacket: 800 unit Suits: 440.8 unit
	Left capacity of BN1(min.)	2400 – 282 = 2118	
	Required capacity for 800 unit jackets(min.)	800 × 1.54 = 1232	
	Left capacity of BN1 to produce jacket(min.)	2118 – 1232 = 886	
	Suit Quantity(unit)	886/2.01 = 440.8	
Scenario 3			
Main Constraint: BN2 Priority: 1. Jackets 2. Suit 3. Trousers	Required capacity for 800 unit Jackets (min.)	800 * 1.26 = 1008	Product Mix Jacket: 800 unit Suits: 679 unit Trousers: 0
	Left capacity of BN2 for suit (min.)	2400 – 1008 = 1392	
	Suit quantity (unit)	1392 / 2.05 = 679.0	

products based on their priorities is determined concerning the available capacity of the main constraint and demand of each product.

In multi constraints environment, all constraints should be eliminated, and then total throughput and net profit should be calculated. From this viewpoint, three scenarios were evaluated if all constraints are eliminated or not with the determined product mix. Evaluations of three scenarios are presented in table 8.

In table 8, according to the determined product mix, the required capacity is calculated for three constraints to understand if constraints remain as constraints or not. For Scenario 1, the product mix is determined as 600 Trousers, 700 Suits, and 461.7 Jackets. Based on Scenario 1, BN1 and BN3 are not constraints because their loads are lower than 2400. BN2 is still a constraint because its load is higher than its capacity (2772.7 > 2400). It needs 372.7 minutes of extra capacity. Therefore, total throughput does not be calculated.

For Scenario 2, the product mix is determined as 600 Trousers, 800 Jackets, and 440.8 Suits. BN1, BN2, and BN3 are not a constraint because their loads are lower than their capacities. The total throughput for Scenario 2 is calculated as ₺ 858560 (600 * 250 + + 800 * 500 + 440.8 * 750). For Scenario 3, the product mix is determined as 800 Jackets, 679 Suits, and 0 Trousers. We could not plan to produce any

Table 8

COMPARISION OF SCENARIOS BASED ON TOC								
Parameter		**CP_k (min.)**	**L_k (min.)**	**d_k (min.)**	**Capacity Using rate**	**Production Priority**	**Product Mix**	**Total Throughput (₺)**
Scenario 1	BN1*	2400	2400.0	0.0	1.00	1.Trousers	600	N.S.
	BN2	2400	2772.7	–372.7	1.16	2.Suit	700	
	BN3	2400	2212.2	187.8	0.92	3.Jacket	461.7	
Scenario 2	BN1*	2400	2400.0	0.0	1.00	1.Trousers	600	₺ 612080
	BN2	2400	2385.6	14.4	0.99	2.Jacket	800	
	BN3	2400	2175.1	224.9	0.91	3.Suit	440.8	
Scenario 3	BN1	2400	2596.8	–196.8	1.08	1.Jacket	800	N.S.
	BN2*	2400	2400.0	0	1.00	2.Suit	679	
	BN3	2400	2289.4	110.6	0.95	3.Trousers	0	

* Main constraint.

Table 9

DMS' EVALUATIONS											
DM1						**DM2**					
Parameter	BN1	BN2	BN3	Throughput	Weights	Parameter	BN1	BN2	BN3	Throughput	Weights
BN1	1	2	3	2	0.417	BN1	1	1	3	1	0.300
BN2	1/2	1	2	2	0.269	BN2	1	1	3	1	0.300
BN3	1/3	1/2	1	1/2	0.121	BN3	1	1/3	1	1/3	0.100
Throughput	1/2	1/2	2	1	0.193	Throughput	1	1	3	1	0.300
DM3						**DM4**					
Parameter	BN1	BN2	BN3	Throughput	Weights	Parameter	BN1	BN2	BN3	Throughput	Weights
BN1	1	1	3	1	0.296	BN1	1	1	2	1/2	0.233
BN2	1	1	3	1	0.296	BN2	1	1	2	2	0.346
BN3	1/3	1/3	1	1/3	0.099	BN3	1/2	1/2	1	1/2	0.135
Throughput	0	2	3	1	0.310	Throughput	2	1/2	2	1	0.330

Trousers. In this situation, BN1 is still a constraint because its load is higher than its capacity (2596.8 > 2400). It needs 196.8 minutes of extra capacity. Therefore, total throughput does not be calculated.

As a result, the best Scenario is Scenario 2.

The GP solution is as follows: Firstly, we define the four decision-makers (DMs) as the production manager, planning manager, planning chief, and accounting manager of the company. DMs performed pairwise comparisons to decide the importance of bottlenecks and throughput by employing AHP. Pairwise comparisons of bottlenecks and throughput are presented in table 9. The importance of bottlenecks and throughput is determined by the arithmetic mean of four DMs evaluations as W1: 0.312, W2: 0.303, W3: 0.114, W4: 0.283.

The maximum throughput of the system which is the goal of throughput is calculated as ₺749800 according to the $\sum_{k=1}^{3} D_k T_k$ $(600*178+800*375+ + 700*490=749800)$.

Finally, using GP, the mathematical model for the product mix of the textile company is defined as follows:

$$\min Z = 0.312 f_1 + 0.303 f_2 + 0.114 f_3 + 0.283 f_4$$

$$2.01X_1 + 1.54X_2 + 0.47X_3 + f_1 = 2400 \quad \text{(for slaughterhouse)}$$

$$2.05X_1 + 1.26X_2 + 0.79X_3 + f_2 = 2400 \quad \text{(for model preparation)}$$

$$1.84X_1 + 1.3X_2 + 0.54X_3 + f_3 = 2400 \quad \text{(for ironing)}$$

$$480X_1 + 367X_2 + 178X_3 + f_4 = 749800 \quad \text{(for throughput)}$$

$$0.99X_1 + 0.65X_2 + 0.34X_3 \leq 2400 \quad \text{(for fusing/labelling)}$$

$$1.54X_1 + 1.54X_2 + 0X_3 \leq 2400 \quad \text{(for Sewing I)}$$

$$1.23 X_1 + 0X_2 + 1.23X_3 \leq 2400 \quad \text{(for Sewing II)}$$

$$1.5X_1 + 0.95X_2 + 0.55X_3 \leq 2400 \quad \text{(for quality control)}$$

$$1.4X_1 + 0.8X_2 + 0.6X_3 \leq 2400 \quad \text{(for combining/Packaging)}$$

$$0 \leq X_1 \leq 700 \quad \text{(for the demand of Suit)}$$

$$0 \leq X_2 \leq 800 \quad \text{(for the demand of Jacket)}$$

$$0 \leq X_3 \leq 600 \quad \text{(for the demand of Trousers)}$$

The above-mentioned GP Model is solved by GAMS LP Programming Software. Provided results are as follows: The product mix is determined as 600 Trousers, 800 Jackets, and 440.796 Suits. With these results, the total throughput is calculated as ₺ 612080. GP results are the same as the TOC-based Scenario 2's results.

CONCLUSIONS

The current study described a methodology for determining the product mix of a manufacturing system utilizing the TOC approach based on developed scenarios and verifying TOC results with GP. The methodology presents a way for determining the product mix in a multiple bottlenecks environment in the textile industry. The proposed methodology can provide the optimum solution in product mix decisions by improving alternative scenarios provided from the results of the TOC approach and by verifying the results of the TOC approach via GP.

TOC algorithm can provide the best solution for product mix decisions in a single-constraint manufacturing environment. When multiple constraint resources exist, the TOC-based approach could not reach the optimal solution and it ran the risk of becoming infeasible. The current study demonstrated that the TOC had flaws when dealing with multiple-constraint resources. Therefore, we used the TOC algorithm suggested by Tanhaei and Nahavandi [15] for multiple-constraint resources. Firstly, we determined three bottlenecks, but the first and second constraints have close values for their loads. Therefore, we had two alternatives, we determined two production priorities of products based on 1st and 2nd bottlenecks. We calculated throughput per constraint time $(t_{i(BN1)})$ based on BN1 for each product. Values of $T_{2(BN1)}$ and $T_{3(BN1)}$ are very close, therefore we have two alternative production priorities for BN1. Thus, we provided three scenarios for the solution of TOC based product mix problem. As seen in table 8 we evaluated three scenarios and found that Scenario 2 is the optimal scenario, in which BN1 is the main constraint and the product mix is suggested as 600 Trousers, 800 Jacket and 441 Suits with a total throughput of ₺ 612080.

The TOC-based algorithm has several benefits: It removes the need for complicated mathematical expressions and it is easy to understand. However, the result of TOC can be infeasible for a multi-constraint environment. Therefore, to verify the TOC results, we employed a GP approach as also suggested by Tanhaei and Nahavandi [13] for product mix problems in multiple constrains manufacturing environments. In this model, DM can decide about the importance of throughput and Bottleneck priority by considering them in the decision matrix. According to the GP model, the product mix was determined as 600 Trousers, 800 Jacket and 441 Suits with ₺ 612080 total throughput. Both approaches produced the same results.

In this study, we attempted to optimize all constraints simultaneously and together within the 3 scenarios to generate feasible results. For this reason, we evaluated the capacity usage rate of all three constraints. If the capacity usage rate of constraints is higher than 1.00, we didn't calculate the total throughput of a scenario. Therefore, for Scenario 1 and Scenario 3, we could not calculate total throughputs because for Scenario 1, the capacity usage rate of the BN2 constraint is higher than 1.00, and in Scenario 3, the capacity usage rate of the BN1 constraint is higher than 1.00.

If we didn't take into account all constraints simultaneously, the total throughput would be calculated as ₺ 612243.9 for Scenario 1 and ₺ 619520 for Scenario 3. We would choose Scenario 3 because of its highest total throughput value. In this situation, we would make a wrong decision because the demand for constraints would continue higher than their capacities. By the scenario development approach, we make accurate decisions as seen from the results of the GP approach.

With the study, changing the company's current production mix and using the product mix created according to the TOC has resulted in positive results that the company can increase its profitability, continuity and sustainability.

This study has some limitations. One of the limitations of this study is that it is a preliminary study that is conducted in a particular sector. Additional analysis should be performed within a larger scope of sectors. Another limitation of this study is that only two methods, i.e. the TOC approach, and the GP, are implemented for determining the optimum product mix. For tackling such problems, there exist several other approaches in the literature that compare their results with that of the TOC. Data provided in this study can be evaluated by other methods such as genetic algorithm, immune system, simulated annealing, etc. and their results can be compared with the findings of this study.

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